CLD9035 Introduction to Set Theory and Applications

Course description

A set is a collection of objects, considered as a single object. Following a series of breakthroughs in the 19th and 20th century, set theory has evolved into an indispensable tool for the modern mathematician, computer scientist, philosopher, or indeed anyone that needs to engage in rigorous reasoning about different classes of objects, whether it be in an ordinary or scientific setting. On one hand, set theory provides an axiomatic foundation for mathematics: proceeding only from a precisely formulated collection of set-theoretic axioms and logical rules, the fundamental objects of mathematical thought (relations, functions, numbers, arithmetical operations) can be defined. On the other hand, set theory provides a rigorous definition of the size of a set, opening the door to a celebrated study of different orders of infinity. In this course, we offer a first introduction to set theory, starting with the naïve laws of the basic set operations, proceeding to set-theoretic definitions of arithmetic and set size, and ending with the axioms of Zermelo-Fraenkel set theory (including the axiom of choice). Previous exposure to university-level logic or mathematics is not required, though a willingness to engage (and develop one's skill) in rigorous logical reasoning and abstract thought is highly recommended.

Instructor

Peter Hawke
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You may email me about course logistics or content. Allow 48 hours for reply.
Regular office hours for meeting students: Tuesdays 11:35-13:35; Wednesdays 13:30-15:30. Or by appointment.

Mode of delivery

The course will be taught in lecture/tutorial format. Lecturing will cover abstract concepts and general techniques, supplemented with concrete examples. Tutorials will be used to discuss exercise questions.

Lectures: Tuesdays 9:30-11:20 at LYH-317.

Tutorials: Wednesdays 12:30-13:20 at LYH-317.

To reduce the chance that you miss emails, all emails sent to you from the instructor will have a subject heading of the form "Set Theory: xxx".

Course website

The course website on Moodle will contain or have a link to:
Class handouts
Assignments
Solutions
Important dates
Readings

Other important information.

Textbook

Tim Button: *Set Theory: An Open Introduction,* The Open Logic Project: https://st.openlogicproject.org

Provisional Course Schedule

Any changes will be announced in seminars and via the course website.

Week	Date	Topics	Textbook Reading
1	13 Jan	Introduction	
PART I: NAÏVE SET THEORY			
2	20 Jan	Logic Preliminaries	
3	27 Jan	Algebra of Sets	Ch.2 [2.1-2.5]
4	3 Feb	Relations	Ch.3 [3.1-3.5]
	5 Feb	Homework Assignment 1 Released	
5	10 Feb	Functions	Ch.4 [4.1-4.5]
6	24 Feb	The Size of Sets	Ch.5 [5.1-5.8]
7	3 Mar	Arithmetization	Ch.6 [6.1-6.5]
8	10 Mar	Mid-term Exam	
PART II: AXIOMATIC SET THEORY			
9	17 Mar	The Iterative Conception	Ch.8 [8.1-8.5]
10	24 Mar	Steps Towards ZF	Ch.9 [9.1-9.7]
11	31 Mar	Ordinals and Replacement	Ch.10 [10.1-10.3; 10.6-10.8]
	2 Apr	Homework Assignment 2 Released	
12	14 Apr	Foundation	Ch.11 [11.1-11.5]
13	21 Apr		

Learning outcomes

Upon completion of the course, students are expected to

LO1. grasp the key concepts and methods of modern axiomatic set theory;

LO2. be adept at techniques for constructing basic but rigorous proofs based on simple logical rules and rooted in set theoretic axioms;

LO3. state and explain the import of various major theorems that stem from modern set theory;

LO4. explain and reconstruct the main line of reasoning behind the proofs of some major results from set theory.

Measurement of learning outcomes

Students' progress towards the learning outcomes will be measured by

1. Homework (two assignments) which will measure a student's grasp of the concepts and techniques covered in lecture and tutorial (LO1, LO2);
2. a mid-term exam and a final exam which will test both the ability to solve unseen problems using standard techniques and to reproduce the statements and proofs for major theorems (LO1, LO2, LO3, LO4);
3. in-class exercises to test a basic knowledge of concepts and techniques (LO1, LO2, LO3). The in-class exercises will consist of short Moodle quizzes conducted as part of each week's tutorial.

Assessment

In-class exercises 20% (ten short Moodle quizzes, 2% each)

Homework 15% (two assignments, 7.5% each)

Midterm exam 25%

Final exam 40%

Important Notes

(1) Students are expected to spend a total of 9 hours (i.e. 3 hours of class contact and 6 hours of personal study) per week to achieve the course learning outcomes.
(2) Students shall be aware of the University regulations about dishonest practice in course work, tests and examinations, and the possible consequences as stipulated in the Regulations Governing University Examinations. In particular, plagiarism, being a kind of dishonest practice, is "the presentation of another person's work without proper acknowledgement of the source, including exact phrases, or summarized ideas, or even footnotes/citations, whether protected by copyright or not, as the student's own work". Students are required to strictly follow university regulations governing academic integrity and honesty.
(3) Students are required to submit writing assignment(s) using Turnitin.
(4) To enhance students' understanding of plagiarism, a mini-course "Online Tutorial on Plagiarism Awareness" is available on https://pla.ln.edu.hk/
(5) Students must adhere to the University's guidelines and practices when using Generative Artificial Intelligence (AI) tools. The official documents "*Guidelines for Using GAI Tools at Lingnan University*" and "*Best Practices for Ethical and Responsible Use of GAI Tools in Course Assessments*" can be found here: https://www.ln.edu.hk/tlc/generative-artificial-intelligence/gai-guidelines-best-practices.
(6) Students may not use generative AI programs (e.g., ChatGPT) in the preparation of any graded assessment items, such as reflection questions or term papers. Similarly, students may not use Google Translate or other AI-driven translation programs to prepare their work in a language other than English and then translate that work into English for submission. Note further that if professional AI detection software judges that a submitted assessment item has likely been written by an AI program, then the instructor will not accept as an excuse/explanation that grammar-correction software (e.g., Grammarly) is the cause of the detection software's score.